

Amendment to the Claims:

This listing of claims replaces all prior versions, and listings, of claims in the application:

1. (Original) A method for estimating the frequency of a single frequency complex exponential tone in additive Gaussian noise, comprising the steps of:

performing the fast Fourier transform (FFT) on the tone;

estimating the frequency as the frequency corresponding to the largest FFT output coefficient magnitude;

computing a discriminant which is proportional to the frequency error in the initial frequency estimate using modified coefficients of the discrete Fourier transform (DFT) with center frequencies plus one half and minus one half of the FFT bin spacing relative to the initial frequency estimate;

mapping the value of the discriminant into the estimate of the frequency error in the initial frequency estimate using a mathematically derived function;

adding the estimate of the frequency error to the initial frequency estimate to get a first interpolated frequency estimate;

computing a further discriminant which is proportional to the frequency error in the first interpolated frequency estimate using modified coefficients of the discrete Fourier transform (DFT) with center frequencies plus one half and minus one half of the FFT bin spacing relative to the first interpolated frequency estimate;

mapping the value of the further discriminant into the estimate of the frequency error in the first interpolated frequency estimate using the mathematically derived function; and

adding the estimate of the frequency error in the first interpolated frequency estimate to the first interpolated

frequency estimate to get a second interpolated frequency estimate.

2. (Original) The method according to claim 1, wherein the first interpolated frequency estimate is in a region of relatively low noise induced frequency error.

3. (Currently amended) The method according to claim [1 or] 2, wherein the method is implemented in computer hardware and/or computer software.

4. (Currently amended) The method according to [any one of the preceding] claim[s] 1, wherein the method is utilised in communications, signal processing and biomedical applications.

5. (Currently amended) The method according to [any one of the preceding] claim[s] 1, further comprising the steps of:

iteratively deriving an interpolated frequency estimate, and
using the frequency discriminant, to obtain a more precise frequency estimate.

6. (Original) The method according to claim 5, wherein the steps of iteratively deriving an interpolated frequency estimate and using the frequency discriminant are repeated until a fixed point solution occurs, where at this fixed point, the discriminant function has zero value.

7. (Currently amended) The method according to [any one of the preceding] claim[s] 1, wherein the frequency discriminant is computed by:

$$D(\varepsilon, \hat{\varepsilon}) = \frac{|\beta| - |\alpha|}{|\beta| + |\alpha|}$$

where, $\varepsilon = fT_s - \frac{k_{\max}}{N}$,

$\hat{\varepsilon} = \hat{f}T_s - \frac{k_{\max}}{N}$, and

β and α are the modified DFT coefficients defined by,

$$\beta = Y(k_{\max} + \frac{1}{2}) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{\max} + \frac{1}{2})}{N}}$$

$$\text{and, } \alpha = Y(k_{\max} - \frac{1}{2}) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{\max} - \frac{1}{2})}{N}}$$

thus, the initial frequency estimate using the FFT, $\hat{f}_0 T_s = \frac{k_{\max}}{N}$ and $\hat{\varepsilon} = 0$.

8. (Currently amended) The method according to [any one of] claim[s] 1 [to 6], wherein the frequency discriminant is computed by:

$$D = \frac{1}{\gamma} \frac{|\beta|^\gamma - |\alpha|^\gamma}{|\beta|^\gamma + |\alpha|^\gamma}, \text{ for } \gamma > 0.,$$

where β and α are the modified DFT coefficients defined by,

$$\beta = Y(k_{\max} + \frac{1}{2}) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{\max} + \frac{1}{2})}{N}}$$

$$\text{and, } \alpha = Y(k_{\max} - \frac{1}{2}) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{\max} - \frac{1}{2})}{N}}$$

9. (Currently amended) The method according to [any one of] claim[s] 1 [to 6], wherein the discriminant of frequency estimation error is computed by:

$$D = \frac{1}{2} \frac{|\beta|^2 - |\alpha|^2}{|\beta|^2 + |\alpha|^2},$$

where β and α are the modified DFT coefficients defined by,

$$\beta = Y(k_{\max} + \frac{1}{2}) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{\max} + \frac{1}{2})}{N}}$$

$$\text{and, } \alpha = Y(k_{\max} - \frac{1}{2}) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{\max} - \frac{1}{2})}{N}}$$

10. (Currently amended) The method according to [any one of] claim[s] 1 [to 6], wherein the frequency discriminant is computed by:

$$D = \text{Re}[\frac{\beta - \alpha^*}{\beta + \alpha^*}]$$

where $\text{Re}[\cdot]$ is the real part and $*$ denotes the complex conjugate, and β and α are the modified DFT coefficients defined by,

$$\beta = Y(k_{\max} + \frac{1}{2}) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{\max} + \frac{1}{2})}{N}}$$

$$\text{and, } \alpha = Y(k_{\max} - \frac{1}{2}) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi n \frac{(k_{\max} - \frac{1}{2})}{N}}$$

11. (Currently amended) The method according to [any one of preceding] claim[s] 1, wherein the frequency discriminant is computed by using more than two DFT coefficients.

12. (Currently amended) The method according to claim 11, wherein $2M+2$ coefficients are used, where $0 \leq M \leq \frac{N}{2}-1$ and the FFT coefficients are used in the frequency discriminant with optimal weighting coefficients obtained by using the concept of matched filtering is,

$$D = \text{Re} \left[\frac{\sum_{m=0}^M C_{[k_{\max} + \frac{1}{2} + m]} \{Y[(k_{\max} + \frac{1}{2} + m) \bmod N] - Y^*[(k_{\max} - \frac{1}{2} - m) \bmod N]\}}{\sum_{m=0}^M \{C_{[k_{\max} + \frac{1}{2} + m]} \{Y[(k_{\max} + \frac{1}{2} + m) \bmod N] + Y^*[(k_{\max} - \frac{1}{2} - m) \bmod N]\}\}} \right]$$

where, $0 \leq M \leq \frac{N}{2} - 1$, $\bmod N$ indicates modulo N ,
and, where, $*$ denotes complex conjugate.

$$C_{k_{\max} + \frac{1}{2} + m} = \frac{e^{j\pi[\frac{1}{2} + m][\frac{N-1}{N}]} \sin[\pi(\frac{1}{2} + m)]}{\sin[\frac{\pi}{N}(\frac{1}{2} + m)]}$$

and, $Y(k_{\max} + \frac{1}{2} + m)$ and $Y(k_{\max} - \frac{1}{2} - m)$ are the modified DFT coefficients given by,

$$Y(k_{\max} + \frac{1}{2} + m) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi m \frac{(k_{\max} + \frac{1}{2} + m)}{N}}$$

$$\text{and, } Y(k_{\max} - \frac{1}{2} - m) = \sum_{n=0}^{N-1} r(n) e^{-j2\pi m \frac{(k_{\max} - \frac{1}{2} - m)}{N}}$$

13. (Currently amended) The method according to claim [11 or] 12, wherein the frequency discriminant using more than two DFT coefficients [us] is used in the last iteration to obtain additional frequency accuracy.

14. (Currently amended) The method according to [any one of] claim[s] 11 [to 13], wherein the frequency discriminant is computed by using more than two DFT coefficients and less or equal to all N FFT coefficients.

15. (Currently amended) The method according to [any one of the preceding] claim[s] 1, wherein additional frequency accuracy

is obtained by computing the frequency discriminant recursively until convergence for the frequency estimate is reached.

16. (Original) The method according to claim 15, wherein convergence for the frequency estimate is reached after zero to three iterations, the number of iterations being dependent on the specific discriminant used and the signal to noise ratio.

17. (Currently amended) The method according to claim [15 or] 16, wherein in any iteration, the frequency discriminant is computed using any one of the functional forms:

$$\Delta f_m(r) = \frac{1}{\pi} \tan^{-1} \left[\frac{|\beta_m| - |\alpha_m|}{|\beta_m| + |\alpha_m|} \tan\left(\frac{\pi}{2N}\right) \right] f_s, \text{ or}$$

$$\Delta f_m(r) = \frac{1}{2N\gamma_m} \left[\frac{|\beta_m|^{\gamma_m} - |\alpha_m|^{\gamma_m}}{|\beta_m|^{\gamma_m} + |\alpha_m|^{\gamma_m}} \right] f_s, \text{ where } \gamma_m \text{ is a constant, } \gamma_m > 0, \text{ or}$$

$$\Delta f_m(r) = \frac{1}{2N} \left[\frac{|\beta_m| - |\alpha_m|}{|\beta_m| + |\alpha_m|} \right] f_s, \text{ for } \gamma = 1, \text{ or}$$

$$\Delta f_m(r) = \frac{1}{4N} \left[\frac{|\beta_m|^2 - |\alpha_m|^2}{|\beta_m|^2 + |\alpha_m|^2} \right] f_s, \text{ for } \gamma = 2.$$

18. (Original) The method according to claim 17, wherein \square varies on each iteration.

19. (Currently amended) The method according to claim [15 or] 16, wherein in any iteration, the frequency discriminant is computed using:

$$\Delta f_m(r) = \frac{1}{2N} \operatorname{Re} \left[\frac{\beta_m - \alpha_m^*}{\beta_m + \alpha_m^*} \right] f_s,$$

where, $\operatorname{Re}[\cdot]$ denotes the real part and $*$ denotes the complex conjugate.

20. (Currently amended) The method according to [any one of] claim[s] 17 [to 19], wherein the frequency incremental

shift, $\Delta f_m(r)$, is related to the previously defined frequency discriminant, D , by,

$$\Delta f_m(r) = \frac{f_s}{2N} D$$

21. (Currently amended) The method according to [any one of] claim[s] 15 [to 20], wherein the frequency discriminant is driven to zero input and output values by either modifying the frequency of the DFT coefficients or frequency translating the signal.

22. (Currently amended) The method according to [any one of] claim[s] 15 [to 21], wherein signal frequency translation is achieved by multiplication of the signal by a locally generated complex exponential signal.

23. (Original) The method according to claim 22, wherein frequency multiplication of the signal is implemented with a standard hardware, software, or combination hardware/software FFT.

24. (Original) The method according to claim 23, wherein the hardware/software FFT is highly optimized for at least one processor operating as a system.

25. (Currently amended) The method according to [any one of] claim[s] 15 [to 24], further comprising the step of scaling the frequency estimate during recursion, to save multiplies.

26. (Original) The method according to claim 25, further comprising a final step of multiplying the scaled frequency

estimate $\hat{f}_{m+1}T_s$ with the sampling frequency f_s to remove the scaling from the frequency estimate.

27. (Currently amended) A frequency estimation software program for estimating the frequency of a single frequency complex exponential tone in additive Gaussian noise, wherein the frequency estimation program has functionality to perform the method according to [any one of] claim[s] 1 [to 26].

28. (Currently amended) A computer system programmed to perform the method according to [any one of] claim[s] 1 [to 26].

29. (Currently amended) The computer system according to claim 28, wherein the hardware includes a DSP processor chip.